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THE EFFECT OF MOLECULAR–KINETIC RESISTANCES ON HEAT-TRANSFER IN CONDENSATION

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Abstract—The usually neglected temperature drop due to the molecular–kinetic mass transfer in change of phase is taken into account in this analysis of Nusselt-type condensation of vapours on a flat vertical plate. Calculations show that the phenomenon has a significant influence at low vapour pressures.

NOMENCLATURE

A,	area;
A_1, A_2,	constants, equations (2.6) and (2.7);
a,	accommodation coefficient;
c', c'',	vapour concentrations at temperatures T' and T'', respectively;
d,	tube diameter;
g,	acceleration due to gravity;
Δh,	enthalpy of vaporization;
M,	molecular weight;
m,	mass;
$\dot{m}$	mass rate;
$\dot{N}$,	rate of molecular evaporation/condensation;
$\tilde{N}$,	Avogadro number;
$(Nu)_m$,	Nusselt modulus;
p,	pressure;
q,	heat flux;
R,	gas constant;
T,	absolute temperature;
x	co-ordinate.

Greek symbols

α,	heat-transfer coefficient;
β,	molecular–kinetic heat-transfer coefficient, equation (1.9);
δ,	film thickness;
λ,	heat conductivity;
ν,	kinematic viscosity;
ρ,	mass density.

Subscript

w,	wall.

Superscripts

$'$,	gas properties at temperature of the liquid;
$''$,	gas properties at temperature of the gas.

1. MOLECULAR–KINETIC MASS AND HEAT TRANSFER ON A LIQUID–GAS BOUNDARY

CONSIDER a flat liquid–gas boundary as shown in Fig. 1. The rate of evaporation of molecules is determined from the kinetic theory (see reference 1), namely

$$\frac{\dot{N}''}{A} = a\tilde{N}c'' \sqrt{\left(\frac{RT''}{2\pi M}\right)}, \tag{1.1}$$

that is, the rate of evaporation is proportional to the concentration of the gas. In conditions of equilibrium the rate of condensing molecules $\dot{N}'/A$ is the same, and $T' = T''$. It follows thus, that

$$\frac{\dot{N}'}{A} = a\tilde{N}c' \sqrt{\left(\frac{RT'}{2\pi M}\right)} \tag{1.2}$$

The coefficient of accommodation, a, appearing in these equations, defines the number of molecules which are adsorbed by the surface in the process of condensation, whereas $(1 - a)$ is the fraction of molecules reflected.

If $T' \neq T''$ the resulting molecule–transfer rate, directed to the liquid, can be evaluated as difference

$$\frac{\dot{N}}{A} = \frac{\dot{N}' - \dot{N}''}{A}.$$

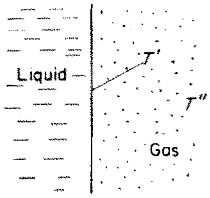

FIG. 1.

Using the gas law

$$c' = p'/(RT'), \quad c'' = p''/(RT'')$$

and taking into account that on a flat phase boundary there is $p' = p'' = p$, we obtain

$$\frac{\dot{N}}{A} = \frac{a\tilde{N}p}{\sqrt{(2\pi MRT'')}}\left(\sqrt{\frac{T''}{T'}} - 1\right). \qquad (1.3)$$

For small temperature differences one may assume

$$\sqrt{\frac{T''}{T'}} - 1 = \sqrt{\left(1 + \frac{T'' - T'}{T'}\right)} - 1 \approx \frac{T'' - T'}{2T''}. \qquad (1.4)$$

Thus

$$\frac{\dot{N}}{A} = \frac{a\tilde{N}p(T'' - T')}{2T''\sqrt{(2\pi MRT'')}} \qquad (1.5)$$

and

$$\frac{\dot{m}}{A} = \frac{a\rho''(T'' - T')}{2} \cdot \sqrt{\left(\frac{R}{2\pi MT''}\right)}. \qquad (1.6)$$

If Δh is the enthalpy of vaporization one may evaluate the connected heat flux

$$q = \Delta h \,.\, \dot{m}/A, \qquad (1.7)$$

or

$$q = \beta(T'' - T'), \qquad (1.8)$$

where

$$\beta = \tfrac{1}{2}\, a\rho''\,\Delta h\sqrt{\left(\frac{R}{2\pi MT''}\right)}. \qquad (1.9)$$

The function $\beta(p)$ for water and $a = 1$ is shown in Fig. 2. It can be expressed by an approximate formula

$$\beta = 2{\cdot}466 \times 10^5 \times p^{0{\cdot}8824} \times a\,\frac{\text{kcal}}{m^2h\ \text{degC}} \qquad (1.10)$$

where p is in technical atmospheres.

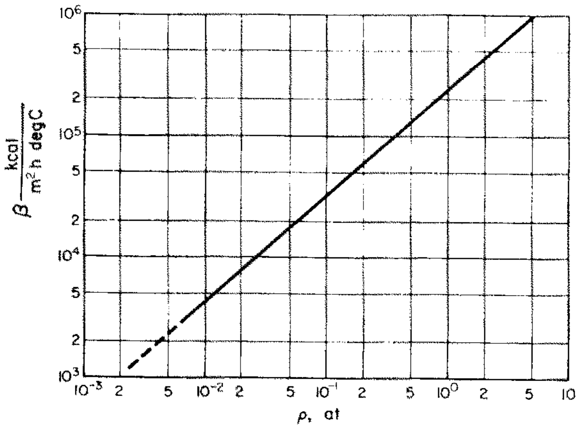

FIG. 2.

2. NUSSELT-TYPE FILM CONDENSATION HEAT TRANSFER

We analyse the process of laminar film condensation on a flat vertical plate. Taking into account all simplifications, due to Nusselt [2], we obtain the expression for heat flux

$$q = \frac{g\rho\,\Delta h}{\nu}\,\delta^2\,\frac{d\delta}{dx} \qquad (2.1)$$

as connected with the growth of film thickness δ, see Fig. 3. On the other hand

$$q = \frac{\lambda}{\delta}(T' - T_w) = \beta(T'' - T'), \qquad (2.2)$$

or

$$q = \frac{\lambda\Delta T}{\delta + \lambda/\beta}, \qquad (2.3)$$

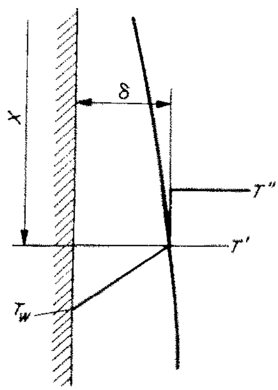

FIG. 3.

where

$$\Delta T = T'' - T_w \tag{2.4}$$

The greater the ratio λ/β, the greater is the effect of molecular-kinetic heat transfer on the phenomenon of condensation. From Fig. 4 it can be seen that the ratio λ/β for water, expressed in metres, is of the order 10^{-4} for lower temperatures and pressures, if the accommodation coefficient $a = 1$. Since film thicknesses at condensation of steam are of that order, the effect of molecular-kinetic resistances may be significant in condensation of steam under vacuum.

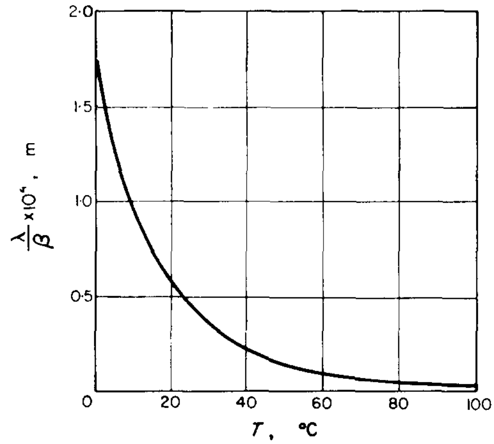

FIG. 4.

Equating the right-hand sides of equations (2.1) and (2.3) we get

$$\frac{g\rho\,\Delta h}{\nu}\,\delta^2\,\frac{d\delta}{dx} = \frac{\lambda\Delta T}{\delta + \lambda/\beta},$$

which yields after integration

$$\delta^4 + A_2\delta^3 = x/A_1\,, \tag{2.5}$$

where

$$A_1 = \frac{g\rho\,\Delta h}{4\nu\lambda\Delta T}, \tag{2.6}$$

$$A_2 = \frac{4}{3}\,\frac{\lambda}{\beta}\,. \tag{2.7}$$

The usual condition $\delta = 0$ at $x = 0$ was taken into account.

The solution of the algebraic equation (2.5), $\delta(A_2, x/A_1)$ is known (see [2] and [3]). We may therefore evaluate the local heat-transfer coefficient

$$a = \frac{\lambda}{\delta + \frac{3}{4}A_2} \tag{2.8}$$

and its mean value

$$a_m = \frac{1}{x}\int_0^x \frac{\lambda dx}{\delta + \frac{3}{4}A_2}, \tag{2.9}$$

or the Nusselt modulus

$$(Nu)_m = \frac{a_m x}{\lambda} = \int_0^x \frac{dx}{\delta + \frac{3}{4}A_2}. \tag{2.10}$$

Now let us introduce the dimensionless quantities

$$u = A_2\left(\frac{A_1}{x}\right)^{\frac{1}{4}}, \quad v = \delta\left(\frac{A_1}{x}\right)^{\frac{1}{4}}, \tag{2.11}$$

into (2.5); this yields

$$v^4 + uv^3 = 1. \tag{2.12}$$

Since

$$x = \frac{A_2^4 A_1}{u^4}, \quad dx = -\frac{4A_2^4 A_1}{u^5}\,du,$$

we obtain

$$(Nu)_m = \frac{4}{3}A_2^3 A_1 \int_0^\infty \frac{3du}{u^4(v + \frac{3}{4}u)}. \tag{2.13}$$

Substituting

$$u = \frac{1 - v^4}{v^3}, \quad du = -\left(\frac{3}{v^4} + 1\right)d\nu,$$

we get

$$(Nu)_m = \frac{4}{3}A_2^3 A_1 \int_0^v \frac{12\,v^{11}\,dv}{(1 - v^4)^4}, \tag{2.14}$$

or

$$(Nu)_m = \frac{4}{3}A_2^3 A_1 \int_y^1 \frac{3(1 - y)^2\,dy}{y^4}, \tag{2.15}$$

where

$$y = 1 - v^4. \tag{2.16}$$

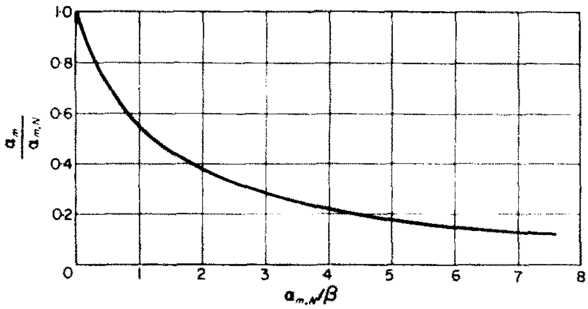

FIG. 5.

Integration of (2.15) yields

$$(Nu)_m = \frac{4}{3} A_2^3 A_1 \left(\frac{1}{y^3} - \frac{3}{y^2} + \frac{3}{y} - 1 \right),$$

or simply

$$(Nu)_m = \frac{4}{3} A_2^3 A_1 \left(\frac{1}{y} - 1 \right)^3. \tag{2.17}$$

From (2.11) and (2.12) it follows that

$$u = A_2 \left(\frac{A_1}{x} \right)^{\frac{1}{4}} = \frac{1 - v^4}{v^3} = \frac{y}{(1-y)^{\frac{3}{4}}}$$

$$= y^{\frac{1}{4}} \left(\frac{1}{y} - 1 \right)^{-\frac{3}{4}},$$

whence

$$\left(\frac{1}{y} - 1 \right)^3 = \frac{yx}{A_2^4 A_1},$$

and

$$(Nu)_m = \frac{4}{3} \frac{yx}{A_2}. \tag{2.18}$$

If $A_2 \to 0$ we obtain the case of Nusselt film condensation and

$$(Nu)_m = (Nu)_{m,N} = \frac{4}{3} A_1 \left(\frac{x}{A_1} \right)^{\frac{3}{4}}. \tag{2.19}$$

Thence equation (2.18) may be written

$$\frac{(Nu)_m}{(Nu)_{m,N}} = \frac{y}{A_2 (A_1/x)^{\frac{1}{4}}}, \tag{2.20}$$

where

$$\lim_{A_2 \to 0} y = A_2 \left(\frac{A_1}{x} \right)^{\frac{1}{4}} = u, \tag{2.21}$$

whereas in general the quantity y satisfies the equation

$$u = \frac{y}{(1-y)^{\frac{3}{4}}}. \tag{2.22}$$

The relationship (2.20) is shown in Fig. 5. Since

$$\frac{(Nu)_m}{(Nu)_{m,N}} = \frac{\alpha_m}{\alpha_{m,N}}, \quad \alpha_{m,N} = \frac{4}{3} \left(\frac{g \rho \lambda^3 \Delta h}{4 \nu \Delta T . x} \right)^{\frac{1}{4}} \tag{2.23}$$

we obtain

$$u = A_2 \left(\frac{A_1}{x} \right)^{\frac{1}{4}} = \frac{\alpha_{m,N}}{\beta}.$$

3. CONCLUSION

As is well-known, the Nusselt formula (2.23) also holds for horizontal tubes if we replace the co-ordinate x by the tube diameter d and change the numerical constant. Therefore the graph in Fig. 5 should hold for horizontal tubes as well. Taking for instance $p = 0{\cdot}0513$ atm, $d = 40$ mm and $\Delta T = 20$ degC we obtain for a steam condenser $\alpha_{m,N} = 6150$ kcal/m²h degC, whereas $\beta = 18\,150$ kcal/m²h degC at $a = 1$. Thence $\alpha_{m,N}/\beta = 0{\cdot}339$, and $\alpha_m/\alpha_{m,N} = 0{\cdot}76$. For the same conditions except of $\Delta T = 2$ degC we obtain $\alpha_m/\alpha_{m,N} = 0{\cdot}67$; and for $\Delta T = 2$ degC and $d = 4$ mm it is $\alpha_m/\alpha_{m,N} = 0{\cdot}54$. These examples show that the influence of molecular–kinetic resistances in vacuum steam condensers may be significant.

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Résumé—La chute de température due au transport de masse au cours d'un changement de phase est habituellement négligée. Elle est prise en considération dans cette analyse de la condensation de vapeurs sur une plaque plane verticale du type étudié par Nusselt. Les calculs montrent que ce phénomène a une influence sensible pour les faibles pressions de vapeur.

Zusammenfassung—Der gewöhnlich vernachlässigte Temperaturabfall infolge des molekularkinetischen Stofftransports bei Phasenänderungen wird in dieser Analyse der Nusselt'schen Wasserhautkondensation von Dampf an einer ebenen senkrechten Platte berücksichtigt. Die Rechnungen ergeben einen ausgeprägten Einfluss des Phänomens bei kleinen Dampfdrücken.

Аннотация—При анализе по Нуссельту конденсации паров на плоской вертикальной пластине учитывался температурный напор за счет молекулярно-кинетического массообмена при фазовом переходе, которым обычно пренебрегают. Расчеты показывают, что это явление оказывает значительное влияние при низких давлениях пара.